D’YUAMEL INTEGRALI HAQIDA

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Maqolada bir jinsli bo‘lmagan to‘lqin tenglamasi uchun Koshi masalasining yechimi bo‘lmish D’yuamel integrali bevosita usulda hosil qilingan.

Kalit so‘zlar: *D’yuamel integrali, Koshi masalasi, bir jinsli bo‘lmagan to‘lqin tenglamasi*

В статье представлен непосредственный метод получения интеграла Дьюамеля, являющийя решением задачи Коши для неоднородного уравнения колебания струны.

Ключевые слова: *интеграл Дьюамеля, задача Коши, неоднородное уравнение колебания струны*

This article discusses the ways of direct creating Dyamel integral which is the settlement of hetregeneous wave equation of Koshi problem.

Key words: *integral of Duamel, problem Cauchi, heterogeneous equqtion.*

1. Masalaning qo‘yilishi.

Matematik fizikaning quyidagi klassik masalasi – bir jinsli bo‘lmagan to‘lqin tenglamasi uchun Koshi masalasini qaraylik:

$$u_{tt}=a^2u_{xx}+f(x,\ t),\ -\infty<x<\infty,\ t>0, \tag{1}$$

$$\begin{cases} u(x,0)=((x) \\ u_t(x,0)=((x) \end{cases} \quad -\infty<x<\infty \tag{2}$$

Odatda, ushbu masala ikkita sodda masalaga ajratib yechiladi:

$$1)\ u_{tt}=a^2u_{xx},\ -\infty<x<\infty,\ t>0, \tag{3}$$

$$\begin{cases} u(x,0)=((x) \\ u_t(x,0)=((x) \end{cases} \quad -\infty<x<\infty \tag{4}$$

$$2)\ u_{tt}=a^2u_{xx}+f(x,\ t),\ -\infty<x<\infty,\ t>0, \tag{5}$$

$$\begin{cases} u(x,0)=0 \\ u_t(x,0)=0 \end{cases} \quad -\infty<x<\infty \tag{6}$$

(3), (4) masalaning yechimi Dalamber formulasi orqali beriladi:

$$u_t(x,t)=\frac{((x+at)+((x-at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}((s)ds \tag{7}$$

(5), (6) masalaning yechimi esa quyidagi formula bilan beriladi:

$$u_2(x,t)=\frac{1}{2a}\int_0^t d\tau\int_{x-a(t-\tau)}^{x+a(t-\tau)} f(s,\tau)ds \tag{8}$$

va dastlabki masalaning yechimi ikkala yechim yig‘indisidan iborat bo‘ladi. Odatda (8) formula D’yuamel integrali deb ataladi.

D’yuamel integrali [1.205], [2.142], [3.58], [4.518], [5.86] da ixtiyoriy o‘zgarmasni variatsiyalash usulining analogi – D’yuamel prinsipi yordamida keltirib chiqarilgan ([3.75] da integral formadagi to‘lqin tenglamasi uchun Koshi masalasi qaraladi va uni yechish jarayonida ham (8) integral hosil qilinadi. [6.252, 268] da fizik mulohazalar yordamida topilgan, [7.227] da umumlashgan funksiyalar nazariyasiga tayanib keltirib chiqarilgan, [8.57] da esa yechim yo‘l bo‘yicha integral ko‘rinishda berilgan.

Mazkur ishda esa D’yuamel integrali matematik tahlilning sodda usullariga tayanib keltirib chiqarilgan. Unda dastlab (8) dagi kabi yondashilgan, keyin esa hosil bo‘lgan karrali integral o‘zgaruvchilarni almashtirish yo‘li bilan soddalashtirilgan va natijada D’yuamel integrali hosil bo‘lgan.

2. D‘yuamel integralini qurish. Ma’lumki,

$$u_{tt}\ =\ a^2u_{xx}\ +\ f(x,t)$$

tenglama giperbolik tipdagi tenglama. Uni kanonik ko‘rinishga keltiramiz, buning uchun

$$\begin{cases} \xi=x+at \\ \eta=x-at \end{cases},\ a>0,\ t\ge 0$$

almashtirish bajariladi. Natijada

$$u_{\xi n}(\xi,\eta)=f\left(\frac{\xi+\eta}{2};\frac{\xi-\eta}{2a}\right)$$

kanonik shakl hosil bo‘ladi. Bu tenglamaning umumiy yechimini topish uchun

$$u_{\xi n}=z(\xi,\eta) \tag{9}$$

belgilash yordamida yangi noma’lum funksiya *z* ni kiritamiz. Shunda

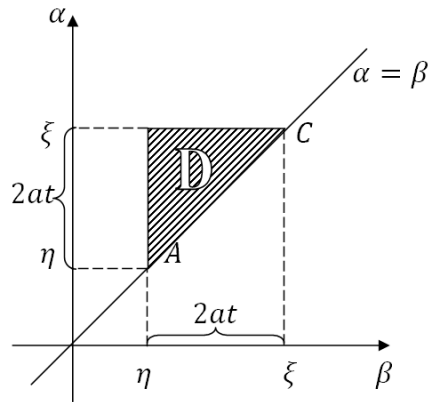
$$z_{\eta}=f\left(\frac{\xi+\eta}{2};\frac{\xi-\eta}{2a}\right) \tag{10}$$

bo‘ladi. Keyin η ni o‘zgarmas deb hisoblab, (9) ni $[\eta;\xi]$ oraliqda ξ bo‘yicha integrallaymiz, bunda $\xi > \eta$ ekani ravshan,

$$\xi-\eta=x+at-(x-at)=2at>0,$$

$$u(\xi,\eta)=\int_{\eta}^{\xi}z(a;\eta)\,da \tag{11}$$

Bundan ξ bo‘yicha hosila olsak, yana (9) hosil bo‘ladi, bunda η ni o‘zgarmas deb hisoblanadi (yuqori chegarasi o‘zgaruvchi bo‘lgan integraldan yuqori chegara bo‘yicha hosila olish qoidasiga ko‘ra).



1-rasm

Endi (10) ni η bo‘yicha integrallaymiz, bunda ξ ni o‘zgarmas deb hisoblab, $[\eta;\xi]$ oraliqda integrallaymiz:

$$z(\xi,\eta)=\int_{\eta}^{\xi}f\left(\frac{\xi+\beta}{2};\frac{\xi-\beta}{2}\right)d\beta=-\int_{\xi}^{\eta}f\left(\frac{\xi+\beta}{2};\frac{\xi-\beta}{2a}\right)d\beta \tag{12}$$

(12) ni (11) ga qo‘yamiz:

$$u(\xi,\eta)=-\int_{\eta}^{\xi}d\alpha\int_{\alpha}^{\eta}f\left(\frac{\alpha+\beta}{2};\frac{\alpha-\beta}{2a}\right)d\beta$$

yoki

$$u(\xi,\eta)=\int_{\alpha}^{\xi}d\alpha\int_{\eta}^{\alpha}f\left(\frac{\alpha+\beta}{2};\frac{\alpha-\beta}{2a}\right)d\beta$$

Bu yerda $\eta\le\alpha\le\xi$, $\eta\le\beta\le\alpha$ ya'ni $\eta\le\beta\le\alpha\le\xi$

Agar (13) integralda integrallash sohasi $D=\{(\alpha,\beta):\eta\le\beta\le\alpha\le\xi\}$ni $O\beta\alpha$ tekisligida tasvirlaydigan bo‘lsak, 1-rasmda tasvirlangan ABC uchburchak hosil bo‘ladi. (13) yechimini qulayroq ko‘rinishda ifodalash uchun

$$\frac{\alpha+\beta}{2}=s,\ \frac{\alpha-\beta}{2a}=\tau$$

almashtirish bajaramiz. Bu geometrik nuqtayi nazardan $O\beta\alpha$ koordinatalar sistemasidan $O\tau s$ kordinatalar sistemasiga 45° ga burilib o‘tishni anglatadi, bunda s o‘qi marta $\sqrt{2}$, o‘qi τ esa $\sqrt{2}\alpha$ marta o‘zgaradi. Bu almashtirishning yakobiani noldan farqli:

$$\left|\frac{D(s,\tau)}{D(s,\beta)}\right|=\begin{Vmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2a} & -\frac{1}{2a}\end{Vmatrix}=\frac{1}{2a}\neq 0$$

Demak, bu almashtirishga teskari almashtirish mavjud va s,τ lar chiziqli erkli

$$\begin{cases}\alpha=s+a\tau\\ \beta=s-a\tau\end{cases}\text{ yoki }\begin{cases}s=\alpha-a\tau\\ s=\beta+a\tau\end{cases}\qquad(14)$$

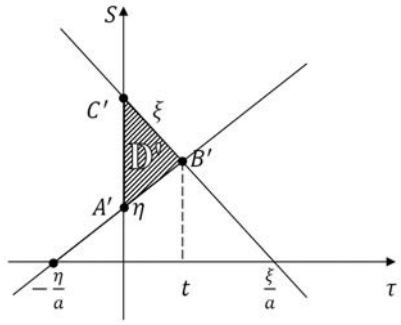
Bu almashtirish natijasida D soha o‘tadigan D' sohani aniqlaymiz:

$$\tau=\frac{\alpha-\beta}{2a}\ge\frac{\beta-\beta}{2a}=0;\ \tau\le\frac{\varsigma-\eta}{2a}=\frac{2at}{2a}=t$$

Demak, $0\le\tau\le t$.

$\alpha\ge\beta$ yarim tekislik (14) ga binoan, $s+\alpha\tau\ge s-\alpha\tau,\ 2\alpha\tau\ge 0$ yoki $a>0$ ekanidan yarim tekislikka o‘tadi.

$\beta=\eta$ chiziq esa yana (14) ga ko‘ra $s=\eta+a\tau$ chiziqqa, $\beta\ge\eta$ yarim tekislik esa $s\ge\eta+a\tau$ yarim-tekislikka o‘tadi.



2-rasm

$\alpha = \xi$ chiziq esa yana (14) ga binoan $s = \xi - at$ chiziqqa, $\alpha \le \xi$ yarimtekislik esa $s \le \xi - at$ yarimtekislikka o‘tadi. Bundan tashqari $s = \eta + at$ va $s \le \xi - at$ chiziqlar $\tau = t$ nuqtada kesishadi (2-rasm).

Demak (14) almashtirish natijasida ABC uchburchak A’B’C’ uchburchakka o‘tadi:

$$D \leftrightarrow D'$$

D' ning uchburchak ekani (14) almashtirishning geometrik ma’nosidan ham kelib chiqadi:

$$D' = \left\{(\tau; s) : 0 \le \tau \le t,\ \eta + a\tau \le s \le \xi - a\tau\right\}$$

Karrali integrallarda o‘zgaruvchilarni almashtirish haqidagi Ostragradskiy formulasiga ko‘ra,

$$\int_{\alpha}^{\xi} d\alpha \int_{\eta}^{\alpha} f\left(\frac{\alpha+\beta}{2}; \frac{\alpha-\beta}{2a}\right) d\beta = \iint_{D} f\left(\frac{\alpha+\beta}{2}; \frac{\alpha-\beta}{2a}\right) dD$$

ni hosil qilamiz. Demak,

$$u_2(\xi, \eta) = \frac{1}{2a} \int_0^t d\tau \int_{\xi+a\tau}^{\xi-a\tau} f(s;\tau)\, ds$$

yoki (x, t) o‘zgaruvchilarda

$$u_2(x,t) = \frac{1}{2a} \int_0^t d\tau \int_{x-a(t-\tau)}^{x+a(t-\tau)} f(s;\tau)\, ds \qquad (7)$$

Tekshirib ko‘rish mumkinki, bu yechim bir jinsli boshlang‘ich shartlarni ham qanoatlantiradi. (7) integral biz keltirib chiqarmoqchi bo‘lgan D‘yuamel formulasidir.

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