IKKINCHI TARTIBLI CHIZIQLI DIFFERENTSIAL TENGLAMA YECHIMINING NOLLARI ORASIDAGI MASOFANI BAHOLASH

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В данной статье оценивается число нулей нетривиального решения линейного дифференциального уравнения второго порядка.

This article deals with the with the number of zeros of nontrivial solutions of the linage differential equations of the second order.

Ushbu maqolada ikkinchi tartibli bir jinsli chiziqli differensial tenglama yechimlarining nollari orasidagi masofalarni baholash masalasini qaraymiz. Bu masala differensial tenglamalar nazariyasining klassik masalalaridan bo'lib, ko'plab ilmiy maqola, darslik va monografiyalarda keltirilgan [1-4]. Ko'plab ma'lumotlarni isbotlari bilan keltirishni lozim topdik.

***1-teorema(Rappoport)**. Aytaylik, $p(t)$ $[0,T]$ kesmada uzluksiz haqiqiy o'zgartuvchili funksiya bo'lsin.*

$$x'' + p(t)x = 0 \quad (1)$$

tenglamaning aynan noldan farqli $x(t)$ yechimining $(0,T)$ yarim ochiq oraliqdagi nollari soni N uchun, ushbu

$$N < \frac{1}{2}\left[T\int_0^T p^+(t)dt \right]^{1/2} + 1 \quad (2)$$

tengsizlik o`rinli, bu yerda $p^+(t) = \max(p(t),0)$.

Isboti. Ma'lumki, agar $p(t)$ $[a,b]$ kesmada uzluksiz haqiqiy funksiya bo`lib, $t = a$ va $t = b$ (1) tenglama $x(t)$ yechimining qo'shni nollari bo'lsa, u holda Lyapunov teoremasiga ko'ra

$$\int_a^b p^+(t)dt > \frac{4}{b-a}$$

tengsizlik o'rinlii. Aytaylik, $x(t_1) = 0$ bo'lib, yechimning qolgan nollari $0 \le t_1 < t_2 < t_3 < \cdots < t_N \le T$ ko'rinishda tartiblangan bo'lsin. U holda $k = 1,2,...,N-1$ uchun Lyapunov teoremasiga ko'ra

$$\int_{t_k}^{t_{k+1}} p^+(t)dt > \frac{4}{t_{k+1} - t_k}$$

Bu tengsizliklarni hadlab qo'shib

$$\int_{t_1}^{t_N} p^+(t)dt > 4\sum_{k=1}^{N-1} \frac{1}{t_{k+1} - t_k}$$

Endi $N-1$ ta sonning o'rta arifmetigi va o'rta garmonigi orasidagi tengsizlikka ko'ra

$$\frac{1}{N-1}\sum_{k=1}^{N-1} \frac{1}{t_{k+1} - t_k} \ge \frac{N-1}{\sum_{k=1}^{N-1}(t_{k+1} - t_k)} = \frac{N-1}{t_N - t_1} \ge \frac{N-1}{T}$$

Shuning uchun

$$\int_{t_1}^{t_N} p^+(t)dt > 4\sum_{k=1}^{N-1} \frac{1}{t_{k+1} - t_k} \ge \frac{4(N-1)^2}{T}$$

bo'lib, isbotlanishi talab qilingan natijaga erishamiz. Teorema isbotlandi.

***2-teorema(Hartman)**. Aytaylik, $p(t)$ $[a,b]$ kesmada uzluksiz haqiqiy funksiya bo`lsin.*

$$x'' + p(t)x = 0 \quad (1)$$

tenglamaning aynan noldan farqli $x(t)$ *yechimining* $(0,T)$ *yarim ochiq oraliqdagi nollari soni* N *uchun, ushbu*

$$N < \int_0^T tp(t)dt + 1 \quad (2a)$$

tengsizlik o`rinli.

Isboti: $\int_a^b m(t)\,p(t)dt > \gamma_m(b-a)$ da $m(t) = t-a$ deb olsak,

$$\gamma = \inf \frac{m(t)}{(t-a)(b-t)} = \inf \frac{t-a}{(t-a)(b-t)} = \inf \frac{1}{b-t} = \frac{1}{b-a}$$

bo`lgani uchun

$$\int_{t_k}^{t_{k+1}} p(t)dt > \frac{4}{t_{k+1}-t_k},\ k = 1,2,...,N-1$$

tengsizliklarga ko`ra, $\int_{t_k}^{t_{k+1}} (t-a)\,p(t)dt > 1$ va o`z navbatida $\int_{t_k}^{t_{k+1}} tp(t)dt > 1$ tengsizlik kelib chiqadi. Shunga ko`ra, $\int_a^b tp(t)dt = \sum_{k=1}^{N-1} \int_{t_k}^{t_{k+1}} tp(t)dt > N-1$ bo`lib, bundan esa (2a) kelib chiqadi. Teorema isbotlandi.

Agar $p(t) = const > 0$ bo`lsa, u holda (1) tenglama $x(t)$ yechimining qo`shni nollari orasidagi masofa $\frac{\pi}{\sqrt{p(t)}}$ ga teng ekanligiga ishonch hosil qilishimiz qiyin emas. Shu sababli (1) tenglamaning aynan noldan farqli $x(t)$ echimining $[0,T]$ kesmadagi nollari soni N bo`lsa, bu echimini kesmaning chetki $t=0$ yoki $t=T$ nuqtalarida nollarga ega bo`lishi yoki bo`lmasligiga qarab ёки $\frac{\pi}{\sqrt{p(t)}} N \le T \le \frac{\pi}{\sqrt{p(t)}}(N+1)$ tengsizliklar bajarilishini ko`rishimiz mumkin.

Аммо $\sqrt{p(t)}T = \int_0^T \sqrt{p(t)}dt$ bo`lgani uchun, yuqoridagi tengsizliklarni umumlashtirib, $\left|\pi N - \int_0^T \sqrt{p(t)}dt\right| \le \pi$ tengsizlikka kelamiz.

Biz bir jinsli ikkinchi tartibli chiziqli

$$y'' + p(x)y' + q(x)y = 0 \quad (3)$$

differensial tenglamani qaraymiz, bu yerda $p(x)$ va $q(x)$ koeffitsientlar $[a,b]$ kesmada berilgan uzluksiz funksiyalardir.

Mavjudlik va yagonalik teoremasiga asosan bu tenglamaning $x_0 \in [a,b]$ nuqtada boshlang'ich qanoatlantiruvchi yechimi yagona bo`lganligi sababli uning aynan noldan farqli ixtiyoriy $y(x)$ yechimi ikki karrali nollarga ega emas, chunki $y(x_0) = y'(x_0) = 0$ shartdan $y(x) = 0$ (1) tenglamaning yechimi ekanligi kelib chiqadi. Shu sababli (1) tenglama $y(x)$ yechimining qo`shni nollari orasidagi masofa musbatdir. Biz qo`shni nollar orasidagi $h$ masofaning musbat quyi chegarasini topish bilan shug`ullanamiz.

3-teorema(Valle—Pussen). Agar qaralayotgan $[a,b]$ kesmada (3) tenglamaning koeffitsientlari

$$|p(x)| \le M,\ |q(x)| \le N \quad (4)$$

tengsizlikni qanoatlantirsa, u holda (3) tenglama ixtiyoriy yechimining qo`shni nollari orasidagi $h$ masofa uchun quyidagi baho o`rinli:

$$h \ge \begin{cases} \dfrac{\sqrt{M^2+2N}-M}{N}, & N \ne 0 \\ \dfrac{1}{M}, & N = 0 \end{cases}$$

Isbot. (3) tenglamaning $[a,b]$ kesmada qaralayotgan ixtiyoriy $y(x)$ yechimining qo`shni nollari $x_1$ va $x_2$ bo`lsin, ya'ni

$$y(x_1) = y(x_2) = 0; \quad x_2 - x_1 = h > 0;$$

U holda, Roll teoremasiga asosan, shunday $\tau \in (x_1, x_2)$ nuqta mavjudki, bu nuqtada $y'(\tau) = 0$ bo'lib

$$y'(x) = y'(\tau) + \int_\tau^x y''(s)ds = \int_\tau^x y''(s)ds$$

tenglik bajariladi. Bundan esa

$$|y'(x)| = \left|\int_\tau^x y''(s)ds\right| \le \int_\tau^x |y''(s)|ds \le \mu(x_2 - x_1) = \mu h,$$

ya'ni

$$|y'(x)| = \mu h \quad (5)$$

bu yerda $\mu = \max\limits_{x_1 \le x \le x_2} |y''(x)|$.

O`z navbatida,

$$y(x) = y(x_1) + \int_{x_1}^x y'(s)ds = \int_{x_1}^x \left(\int_\tau^x y''(s)ds\right)dt = \int_{x_1}^x (x-s)y''(s)ds$$

bo`lgani uchun

$$|y(x)| = \left|\int_{x_1}^x (x-s)y''(s)ds\right| \le \int_{x_1}^x (x-s)|y''(s)|ds \le$$

$$\le \mu\int_{x_1}^x (x-s)ds = \mu\frac{(x-x_1)^2}{2} = \frac{\mu h^2}{2};$$

ya'ni

$$|y(x)| = \frac{\mu h^2}{2}; \quad (6)$$

tengsizlik bajariladi.

Endi (3) tenglamani $-y''(x) = p(x)y'(x) + g(x)y(x)$ ko`rinishida yozib, (4), (5) va (6) munosabatlarga asosan baholaymiz. Natijada,

$$|y''(x)| \le |p(x)y'(x)| + |g(x)y(x)| \le |p(x)| \cdot |y'(x)| + |g(x)| \cdot |y(x)| \le M\mu h + N\mu\frac{h^2}{2},$$

ya'ni $|y''(x)| \le \mu\left(\dfrac{Nh^2}{2} + Mh\right)$. Endi $\max\limits_{[x_1;x_2]} |y''(x)| = \mu$ ekanligini e'tiborga olib

$$\frac{Nh^2}{2} + Mh - 1 \ge 0 \quad (7)$$

tengsizlikni va bundan o`z navbatida

$$h \ge h_0 = \frac{\sqrt{M^2+2N}-M}{N} \quad (8)$$

tengsizlikni hosil qilamiz. Agar (7) da $N=0$ bo`lsa, $h \ge \frac{1}{M}$ kelib chiqadi. Teorema isbotlandi.

Endi biz (1) tenglama yechimining qo'shni nollari orasidagi masofani baholash uchun Xartman—Uitteker usulini va bu usulni yaxshilash mumkinligini ko`rsatib o`tamiz. Bu usul quyidagicha:

Agar $\varphi(x)$ funksiya $[0,h]$ intervalda uzluksiz differensiallanuvchi bo`lsa, u holda

$$h\varphi(x)=\int_0^x \xi\varphi'(\xi)d\xi-\int_0^h (h-\xi)\varphi'(\xi)d\xi+\int_0^h \varphi(\xi)d\xi \quad (9)$$

ayniyatga asoslangan .

Haqiqatdan bo`laklab integrallash yordamida

$$\int_0^x \xi\varphi'(\xi)d\xi = x\varphi(x)-\int_0^x \varphi(\xi)d\xi$$

$$\int_0^h (h-\xi)\varphi'(\xi)d\xi = -(h-x)\varphi(x)+\int_0^h \varphi(\xi)d\xi$$

tengliklarni hosil qilamiz va bu tengliklarning birinchisidan ikkinchisini ayirib, (9) tenglikni hosil qilamiz.

Aytaylik, (3) tenglamaning qandaydir $y(x)$ yechimi $x=0$ va $x=h$ nuqtalarda ikkita ketma-ket nollarga ega bo`lsin. (9) ayniyatni $\varphi(x)=y'(x)$ funktsiya uchun qo`llaymiz . U holda

$$\int_0^h \varphi(\xi)d\xi=\int_0^h y'(\xi)d\xi=y(h)-y(0)=0$$

ekanligini e'tiborga olinsa, (9)

$$hy'(x)=\int_0^x \xi y''(\xi)d\xi-\int_0^h (h-\xi)y''(\xi)d\xi \quad (10)$$

ko`rinishni oladi. (3) tenglamadan $y''(x)=-p(x)y'(x)-g(x)y(x)$ bo`lgani uchun, (10) tenglikdan

$$\begin{aligned} hy'(x) = &-\int_0^x \xi p(\xi)y'(\xi)d\xi+\int_0^h (h-\xi)p(\xi)y'(\xi)d\xi- \\ &-\int_0^x \xi q(\xi)y(\xi)d\xi+\int_0^h (h-\xi)q(\xi)y(\xi)d\xi \end{aligned} \quad (11)$$

munosabat hosil bo'ladi.

$\mu=\max_{[0,h]}|y'(x)|$ belgilash kiritamiz. $y(0)=y(h)=0$ bo`lgani uchun $y(x)=\int_0^x y'(\xi)d\xi$ ва $y(x)=-\int_0^h y'(\xi)d\xi$ tengliklardan mos ravishda hosil qilinadigan $|y(\xi)|\le \mu x,\ |y(x)|\le \mu(h-x)$ tengsizliklar $[0,h]$ kesmaning barcha nuqtalarida bir vaqtda bajarilishini ko`ramiz.

Demak,

$$|y(\xi)|\le \mu\min\{\xi,h-\xi\} \quad (12)$$

va (12) ni hisobga olgan holda (11) ni baholaymiz.

$$h|y'(x)|\le M\int_0^x \xi|y(\xi)|d\xi+N\int_0^x (h-\xi)|y(\xi)|d\xi \le \mu M\frac{\xi^2+(h-\xi)^2}{2}+$$

$$+\mu N\left[\int_0^x \xi\min\{\xi,h-\xi\}d\xi+\int_0^h (h-\xi)\min\{\xi,h-\xi\}d\xi\right]$$

Oxirgi tengsizlikda

$$y'(x)\le\mu \text{ ва } \frac{x^2+(h-x)^2}{2}\le\frac{h^2}{2}$$

ekanligi e'tiborga olinsa, bu tengsizlik

$$1 \le \frac{Mh}{2} + \frac{N}{h} \cdot F(h) \quad (13)$$

ko`rinishda ifodalanishi mumkin, bu yerda

$$F(h) = \int_0^{h/2} \xi \min\{\xi, h-\xi\} d\xi + \int_{h/2}^{h} (h-\xi) \min\{\xi, h-\xi\} d\xi$$

Endi biz $\min\{\xi, h-\xi\}$ ning mumkin bo`lgan barcha qiymatlari bo`yicha $F(h)$ ni baholaymiz. Quyidagi hollarni qaraymiz:

1-hol.

$$\min\{\xi, h-\xi\} = \begin{cases} \xi \quad agar \quad 0 \le \xi \le \dfrac{h}{2} \quad bo'lsa, \\ h-\xi \quad agar \quad \dfrac{h}{2} < \xi < h \quad bo'lsa \end{cases}$$

bo`lsin. Bu holda

$$F(h,x) = \int_0^{h/2} \xi^2 d\xi + \int_{h/2}^{h} (h-\xi)^2 d\xi = \frac{h^3}{24} + \frac{h^3}{24} = \frac{h^3}{12}$$

bo`lib, o`ng tomonining eng katta qiymati $\xi = \dfrac{h}{2}$ da erishiladi, ya'ni $F(h) \le \dfrac{h^3}{12}$. Natijada, (8) dan

$$\frac{Nh^2}{12} + \frac{Mh}{2} - 1 \ge 0$$

tengsizlikni va bundan h masofa uchun

$$h \ge h_1 = \begin{cases} \dfrac{\sqrt{9M^2 + 12N^2} - 3M}{N}, & agar\ N \ne 0 \quad bo'lsa, \\ \dfrac{2}{M}, & agar\ N = 0 \quad bo'lsa \end{cases}$$

bahoni hosil qilamiz.

2-hol.

$$\min\{\xi, h-\xi\} = \begin{cases} h-\xi \quad agar \quad 0 \le \xi \le x \quad bo'lsa, \\ \xi \quad agar \quad x < \xi < h \quad bo'lsa \end{cases}$$

bo`lsin. Bu holda

$$F(h,x) = \int_0^{x} \xi(h-\xi) d\xi + \int_x^{h} (h-\xi)\xi d\xi = \int_0^{h} (h-\xi)\xi d\xi = \frac{h^3}{6}$$

bo`lib, (8) dan $\dfrac{Nh^2}{6} + \dfrac{Mh}{2} - 1 \ge 0$ tengsizlikni va bundan h masofa uchun

$$h \ge h_2 = \begin{cases} \dfrac{\sqrt{9M^2 + 24N^2} - 3M}{2N}, & agar\ N \ne 0 \quad bo'lsa, \\ \dfrac{2}{M}, & agar\ N = 0 \quad bo'lsa \end{cases}$$

bahoni hosil qilamiz.

3-hol.

$$\min\{\xi, h-\xi\} = \xi,\ 0 < x < h$$

bo`lsin. Bu holda

$$F(h,x) = \int_0^x \xi^2 d\xi + \int_x^h (h-\xi)\xi d\xi = \frac{h^3 + 4x^3 - 3hx^2}{6},\ F(h,x) \le \frac{h^3}{8}.$$

Natijada, (8) dan

$$\frac{Nh^2}{8} + \frac{Mh}{2} - 1 \ge 0$$

tengsizlikni va bundan h masofa uchun

$$h \ge h_3 = \begin{cases} \dfrac{\sqrt{4M^2 + 8N^2} - 2M}{2N}, & agar\ N \ne 0\ \ bo'lsa \\ \dfrac{2}{M}, & agar\ N = 0\ \ bo'lsa \end{cases}$$

bahoni hosil qilamiz.

4-hol.

$\min\{\xi, h-\xi\} = h-\xi,\ 0 < x < h$

bo`lsin. Bu holda

$$F(h,x) = \int_0^x \xi(h-\xi)d\xi + \int_x^h (h-\xi)^2 d\xi = \frac{hx^2}{2} + \frac{(h-x)^3 - x^3}{3}$$

bo`lib, tenglikning o`ng tomoni $\frac{h^3}{8}$ dan ortmaydi. Demak, bu holda ham 3-holdagi baho saqlanadi. Yuqoridagi hollarni taqqoslab,

$$h \ge \max(h_1, h_2, h_3) = h_3 = \begin{cases} \dfrac{\sqrt{4M^2 + 8N^2} - 2M}{2N}, & agar\ N \ne 0\ \ bo'lsa \\ \dfrac{2}{M}, & agar\ N = 0\ \ bo'lsa \end{cases}$$

xulosaga kelamiz.

Eslatma.

1. Xartman—Uitteker usuli bo`yicha $h > h_2$ baho, Mishkis taklifiga ko`ra $h \ge h_3$ baho qayd etilgan.

2. Bu teorema Valle—Pussen teoremasiga nisbatan aniqroqdir, chunki $h_1 > h_2$.

Endi yuqorida ko`rib o`tilgan 3-holning boshqa usulda isbotini keltiramiz. Buning uchun ushbu ayniyatdan foydalanamiz:

Agar $y(x) \in C^2[0,h]$ funktsiya uchun $y(0) = y(h) = 0$ bo`lsa, u holda

$$-hy(x) = (h-x)\int_0^x \xi y''(\xi)d\xi + \int_x^h (h-\xi)y''(\xi)d\xi \qquad (14)$$

ayniyat o`rinli.

Haqiqatan bo`laklab integrallash vositasida

$$(h-x)\int_0^x \xi y''(\xi)d\xi = (h-\xi)\left[\xi y'(\xi)\Big|_0^x - \int_0^x y'(\xi)d\xi\right] =$$
$$= (h-x)[xy'(x) - y(x)] = (h-x)xy'(x) - (h-x)y(x)$$

ya'ni

$$(h-x)\int_0^x \xi y''(\xi)d\xi = (h-x)xy'(x) - (h-x)y(x) \quad (15)$$

tenglikni hosil qilamiz. Xuddi shu kabi

$$x\int_{x}^{h}(h-\xi)y''(\xi)d\xi=-(h-x)xy'(x)-xy(x) \quad (16)$$

tenglikni hosil qilamiz. (15) va (16) tengliklarni hadlab qo`shib, (14) ayniyatni hosil qilamiz. $\max\limits_{0\le x\le h}|y''(x)|=\mu$ belgilash kiritib, (14) dan va uni differensiallash natijasida hosil bo`ladigan

$$hy'(x)=\int_{0}^{x}\xi y''(\xi)d\xi-\int_{x}^{h}(h-\xi)y''(\xi)d\xi \quad (17)$$

tenglik natijasidan foydalanib, $|y(x)|$ va $|y'(x)|$ larni baholaymiz. (17) ayniyatdan:

$$|y(x)|\le\frac{\mu}{h}\left[(h-x)\int_{0}^{x}\xi d\xi+x\int_{x}^{h}(h-\xi)d\xi\right]=\frac{\mu}{2}x(h-x)\le\frac{\mu}{2}\left(\frac{x+h-x}{2}\right)^2=\frac{\mu h^2}{8}$$

tengsizlikni hosil qilamiz. (17) dan esa

$$|y'(x)|\le\frac{\mu}{2h}\left[\int_{0}^{x}\xi d\xi+\int_{x}^{h}(h-\xi)d\xi\right]=\frac{\mu}{2h}[x^2+(h-x)^2].$$

Bu tengsizlik barcha $0\le x\le h$ kesmada o`rinli bo`lganidan

$$|y'(x)|\le\frac{\mu}{2h}\left[h^2+(h-h)^2\right]=\frac{\mu h}{2}$$

tengsizlik kelib chiqadi. Demak,

$$|y'(x)|\le\frac{\mu h^2}{8};\ |y(x)|\le\frac{\mu h}{2} \quad (18)$$

(3) tenglamadan (4) va (18) ni hisobga olgan holda,

$$\mu=\max_{[0,h]}|y''(x)|\le\max_{[0,h]}\left[|p(x)|\cdot|y(x)|+|q(x)|\cdot|y(x)|\right]\le\mu\left(\frac{Mh}{2}+\frac{Nh^2}{8}\right)$$

tengsizlikni hosil qilamiz. Bundan esa $\frac{Nh^2}{8}+\frac{Mh}{2}-1\ge 0$ тенгсизликни hosil qilamiz. Bu tengsizlik faqat musbat yechimga ega, bundan, $h\ge h_3$ bahoni hosil qilamiz.

Endi differensial tenglama yechimining nollari orasidagi masofani baholashga integral tengsizliklarning tatbiqini ko`rib o`tamiz.

Bizga koeffitsientlari (4) tengsizlikni qanoatlantiruvchi (3) differensial tenglama berilgan bo`lsin. Agar $\varphi\in C^2[0,h]$ funksiya $\varphi(0)=\varphi(h)=0$ munosabatni kanoatlantirsa, u holda Virtingerning ushbu tengsizligi o`rinli edi:

$$\int_{0}^{h}\varphi^2(x)dx\le\frac{h^2}{\pi}\int_{0}^{h}\varphi'^2(x)dx \quad (19)$$

Bu tengsizlikdan foydalanib quyidagi teoremani isbotlash mumkin.

4-teorema. Agar (3) tenglamaning koeffitsientlari (4) tengsizlikni qanoatlantirsa, u holda

$$\frac{Nh^2}{\pi^2}+\frac{Mh}{\pi}-1\ge 0 \quad (20)$$

tengsizlik o`rinli.

Isboti. (3) tenglamaning har ikkala qismini $y(x)$ ga ko`paytiramiz va $y(0)=y(h)=0$ ekanligini e'tiborga olgan holda bo`laklab integrallaymiz.

$$-\int_{0}^{h}y''(x)y(x)dx=\int_{0}^{h}p(x)y'(x)y(x)dx+\int_{0}^{h}q(x)y^2(x)dx$$

$$-\int_0^h y''(x)y(x)dx = -\int_0^h y(x)d(y'(x)) = -y(x)y'(x)\Big|_0^h + \int_0^h y'^2(x)dx =$$

$$= -y(h)y'(h) + y(0)y'(0) + \int_0^h y'^2(x)dx = \int_0^h y'^2(x)dx$$

Demak,

$$\int_0^h y'^2(x)dx = \int_0^h p(x)y'(x)y(x)dx + \int_0^h q(x)y^2(x)dx \quad (21)$$

(4) cheklanishlarni hisobga olgan holda, Koshi – Bunyakovskiy va Virtingerning (19) tengsizligidan foydalanib, (21) ni baholaymiz

$$\int_0^h y'^2(x)dx \le \int_0^h |p(x)||y'(x)|\cdot|y(x)|dx + \int_0^h |q(x)|y^2(x)dx \le N\int_0^h y^2(x)dx \le$$

$$\le M\left(\frac{h^2}{\pi^2}\int_0^h y'^2(x)dx\right)^{1/2}\cdot\left(\int_0^h y'^2(x)dx\right)^{1/2} + N\frac{h^2}{\pi^2}\int_0^h y'^2(x)dx = \left(\frac{Nh^2}{\pi^2}+\frac{Mh}{\pi}\right)\int_0^h y'^2(x)dx$$

Bundan (20) kelib chiqadi.

Teorema isbotlandi.

(20) tengsizlikdan $h \ge h_4 = \dfrac{\pi\left(\sqrt{M^2+4N}-M\right)}{2N}$ baho kelib chiqadi. Virtinger tengsizligi bilan bir qatorda $y \in C^1[0,h]$, $y(0) = y(h) = 0$ funksiya uchun Opyalning

$$\int_0^h |y(x)y'(x)|dx \le \frac{h}{4}\int_0^h y'^2(x)dx \quad (22)$$

tengsizligini qo'llab quyidagi natijaga erishamiz.

5-teorema. Agar (3) tenglamaning koeffitsientlari (7) tengsizlikni qanoatlantirsa, u holda (3) tenglama yechimining qo`shni nollari orasidagi h masofa uchun

$$\frac{Nh^2}{\pi^2}+\frac{Mh}{4}-1 \ge 0 \quad (23)$$

tengsizlik o`rinli.

Isboti: (3) tenglamaning aynan noldan farqli $y(x)$ echimi $x=0$ va $x=h$ nuqtalarda qo`shni nollarga ega bo`lsin, ya'ni , $y(0) = y(h) = 0$ ekanligini hisobga olgan holda (6) tenglamaning har ikkala qismini $y(x)$ga ko`paytirib, bo`laklab integrallash yordamida (21) tenglikni hosil qilamiz.

Endi Virtingerning (19), Opyalning (22) tengsizliklaridan foydalangan holda (21) ni baholaymiz. Natijada,

$$\int_0^h y'^2(x)dx \le \int_0^h |p(x)|\cdot|y'(x)y(x)|dx + \int_0^h q(x)y^2(x)dx \le$$

$$\le M\int_0^h y(x)y'(x)dx + n\int_0^h y^2(x)dx \le \frac{Mh}{4}\int_0^h y'^2(x)dx +$$

$$+ N\frac{h^2}{\pi^2}\int_0^h y'^2(x)dx + \left(\frac{Mh}{4}+\frac{Nh^2}{\pi^2}\right)\int_0^h y'^2(x)dx$$

Bu tengsizlikdan (23) kelib chikadi. Teorema isbotlandi.

(23) ga ko`ra,

$$h \ge h_5 = \frac{\pi\left(\sqrt{M^2\pi^2+16N}-M\pi\right)}{8N}$$

bahoni hosil qilamiz.

Polyak matematigi Opyal tengsizlikni

$$\frac{Nh^2}{\pi^2}+\frac{Mh}{\pi}-1\geq 0 \quad (24)$$

tengsizlikkacha kuchaytirish mumkinligini isbotlagan.

Bu tengsizlik e'tiborga olinsa, (24) dan $h\geq\frac{\sqrt{M^2+N\pi^2}-M}{N}$ baho kelib chiqadi.

Demak, (3) differensial tenglama yechimining qo`shni nollari orasidagi h masofa, tenglama koeffitsientlari (4) shartni qanoatlantirganda hosil bo'ladigan natijalar quyidagi jadvalda keltirilgan.

№		$h(N\neq 0)$	$H(N=0)$
h_0	Valle—Pussen	$\frac{\sqrt{M^2+2N}-M}{N}$	$\frac{1}{M}$
h_1	Mishkis	$\frac{\sqrt{9M^2+12N}-3M}{N}$	$\frac{2}{M}$
h_2	Xarman-Uitner	$\frac{\sqrt{9M^2+24N}-3M}{2N}$	$\frac{2}{M}$
h_3		$\frac{\sqrt{4M^2+8N}-2M}{N}$	$\frac{2}{M}$
h_4		$\frac{\pi(\sqrt{M^2+4N}-M)}{2N}$	$\frac{2}{M}$
h_5	Opyal	$\frac{\pi(\sqrt{M^2\pi^2+16N}-M\pi)}{8N}$	$\frac{2}{M}$

Eslatma. Yuqorida keltirilgan ma'lumotlarni qabariq analiz va optimal boshqaruv nazariyasiga asoslangan holda yanada kengaytirish mumkinligi [5,6] da amalga oshirilgan.

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