INVOLYUTSIYAGA EGA BO'LGAN TENGLAMALAR UCHUN BA'ZI SPEKTRAL MASALALAR

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Maqolada involyutsiya xossasiga ega bo'lgan differensial tenglama uchun yozilgan spektral masalada ba'zi chegaraviy shartlar bo'yicha xos qiymatlar va xos funksiyalarni topishga oid misollar ko'rib chiqilgan.

Kalit so'zlar: *involyutsiya, spektral masala, davriy va antidavriy chegaraviy shartlar.*

В статье рассмотрены примеры нахождения собственных чисел и собственных векторов спектральной задачи дифференциального уравнения с инволюцией при некоторых краевых условиях.

Ключевые слова: *инволюция, спектральная задача, периодическое условие, антипериодическое условие.*

In this paper the examples to obtained eigenvalue and eigenfunctions for spectral problem of differential equation with involution for some boundary conditions have be considered

Key words: *involution, spectral problem, periodic and anti periodic boundary conditions.*

[1-3] adabiyotlarda involyutsiya qatnashgan xususiy hosilali differensial tenglamalar qaralgan bo'lib, chegaraviy masalalarni yechish uchun Fur'ye usulini qo'llash orqali Koshi va spektral masalalarni yechishga olib kelingan. Ammo bu maqolalarda hosil bo'lgan spektral masalalarni yechish jarayoni keltirilmagan. Biz bu maqolada involyutsiyaga ega bo'lgan to'lqin tenglamasi uchun spektral masalani yechish jarayoniga to'xtalamiz.

Aytaylik, $D=\{(x,t): -1<x<1,\quad 0<t<\infty\}$ sohada involyutsiyaga ega bo'lgan xususiy hosilali

$$\frac{\partial u(x,t)}{\partial t}=\frac{\partial^2 u(-x,t)}{\partial x^2}-\alpha\frac{\partial^2 u(x,t)}{\partial x^2} \tag{1}$$

parabolik tenglamaning $C^{2,1}(D)\bigcap C(\overline{D})$ funksiyalar sinfida

$$u(x,0)=\varphi(x),\ \varphi(-1)=\varphi(1)=0 \tag{2}$$

boshlang'ich

$$u(-1,t)=0\ ,\ u(1,t)=0 \tag{3}$$

va

$$u'(-1,t)=0\ ,\ u'(1,t)=0 \tag{4}$$

chegaraviy shartlarni qanoatlantiruvchi yechimini topish masalasi qoyilgan bo'lsin.

O'zgaruvchilarni ajratishning Fur'ye usuli bo'yicha masalalarning izlanayotgan $u(x,t)$ yechimini

$$u(x,t)=X(x)\cdot T(t)$$

ko'rinishda izlaymiz. U holda (1) tenglama

$$X(x)\cdot T'(t)-X''(x)\cdot T(t)+\varepsilon X''(-x)\cdot T(t)=0\,,$$

yoki

$$\frac{T'(t)}{T(t)}=\frac{X''(x)-\varepsilon X''(-x)}{X(x)}$$

ko'rinishni oladi. Yuqoridagi masalaning berilishidan yechim chegaralangan bo'lishi uchun

$$\frac{T'(t)}{T(t)}=\frac{X''(x)-\varepsilon X''(-x)}{X(x)}=-\lambda,\ \lambda>0$$

deb tanlansa, bu tenglikdan $T'(t)+\lambda T(t)=0$ va $X''(x)-\varepsilon X''(-x)+\lambda X(x)=0$ tenglamalarni hosil qilamiz. Masalada berilgan chegaraviy shartlarga hamda masala yechimining $u(x,t)=X(x)\cdot T(t)$ tanlanishiga ko'ra

$$X''(x)-\varepsilon X''(-x)+\lambda X(x)=0, \tag{5}$$

$$X(-1)=X(1)=0 \tag{6}$$

va

$$X''(x)-\varepsilon X''(-x)+\lambda X(x)=0, \tag{7}$$

$$X'(-1)=X'(1)=0 \tag{8}$$

spektral masalalarga ega bo'lamiz.

(5)-(6) spektral masalaning xos qiymati va xos vektorlarini topamiz.

Har qanday funksiyani juft va toq funksiyalar yig'indisi ko'rinishida ifodalash mumkin bo'lgani uchun spektral masalaning izlanayotgan $X(x)$ yechimini

$$X(x)=F(x)+G(x)$$

ko'rinishda izlaymiz, bu yerda $F(-x)=F(x)$ va $G(-x)=-G(x)$.

Natijada (5) tenglama

$$[F(x)+G(x)]''-\varepsilon[F(x)-G(x)]''+\lambda[F(x)+G(x)]=0,$$

yoki

$$-(1-\varepsilon)F''(x)-(\varepsilon+1)G''(x)=\lambda F(x)+\lambda G(x)$$

ko'rinishga keladi. Shuning uchun (5)-(6) spektral masala

$$\begin{cases}(1-\varepsilon)F''(x)+\lambda F(x)=0, & -1\le x\le 1\\ F(-1)=F(1)=0\end{cases} \tag{9}$$

va

$$\begin{cases}(1+\varepsilon)G''(x)+\lambda G(x)=0, & -\pi\le x\le \pi\\ G(-1)=G(1)=0\end{cases} \tag{10}$$

spektral masalalarga tarmoqlanadi.

(9) va (10) spektral masalalarning umumiy yechimlari mos ravishda

$$F(x)=A\cos\sqrt{\frac{\lambda}{1-\varepsilon}}x+B\sin\sqrt{\frac{\lambda}{1-\varepsilon}}x \text{ va } G(x)=C\cos\sqrt{\frac{\lambda}{1+\varepsilon}}x+D\sin\sqrt{\frac{\lambda}{1+\varepsilon}}x$$

ko'rinishga ega, bu yerda A,B,C va D – ixtiyoriy o'zgarmas sonlar. U holda $F(-1)=F(1)=0$ va $G(-1)=G(1)=0$ chegaraviy shartlarga ko'ra, $A=0, D=0$ bo'lgani uchun , (5)-(6) spektral masala

$$\lambda_{k,1}=(1+\varepsilon)k^2\pi^2,\ k\in N, \qquad \lambda_{k,2}=(1-\varepsilon)\left(k+\frac{1}{2}\right)^2\pi^2,\ k\in N\cup\{0\} \tag{11}$$

xos qiymatlarga ega. Bu xos qiymatlarga

$$\chi_{k,1}=\sin kx,\ k\in N, \quad \chi_{k,2}=\cos\left(k+\frac{1}{2}\right)x,\ k\in N\cup\{0\}$$

xos funksiyalar mos keladi. Shu bilan birga

$$\left\|\chi_{k,1}\right\|^2 = \int_{-\pi}^{\pi} \sin^2 kx dx = \frac{1}{2}\int_{-\pi}^{\pi}\left(1-\cos 2kx\right)dx = \pi$$

va

$$\left\|\chi_{k,2}\right\|^2 = \int_{-\pi}^{\pi} \cos^2\left(k+\frac{1}{2}\right)x dx = \frac{1}{2}\int_{-\pi}^{\pi}\left(1+\cos\left(2k+1\right)x\right)dx = \pi$$

bo'lgani uchun (11) xos qiymatlar bo'yicha normalangan xos funksiyalar mos ravishda

$$X_{k,1} = \frac{1}{\sqrt{\pi}}\sin kx,\ k\in N\ ,\quad X_{k,2} = \frac{1}{\sqrt{\pi}}\cos\left(k+\frac{1}{2}\right)x,\ k\in N\bigcup\{0\} \quad (12)$$

ko'rinishga ega bo'ladi.

Xuddi shu kabi, (7)-(8) spektral masalaning yechilishi (5)-(6) spektral masalaning xos qiymati va xos vektorlarini topishga o'xshash bo'lib, uning yechimi

$$\begin{cases}(1-\varepsilon)F''(x)+\lambda F(x)=0,\ -\pi\le x\le\pi\\ F'(-1)=F'(1)=0\end{cases}$$

va

$$\begin{cases}(1+\varepsilon)G''(x)+\lambda G(x)=0,\ -\pi\le x\le\pi\\ G(-1)=G(1)=0\end{cases}$$

spektral masalalar yechimlaridan tashkil topadi.

(7)- (8) spektral masalalarning umumiy yechimlari mos ravishda

$$F(x)=A\cos\sqrt{\frac{\lambda}{1-\varepsilon}}x+B\sin\sqrt{\frac{\lambda}{1-\varepsilon}}x \text{ va } G(x)=C\cos\sqrt{\frac{\lambda}{1+\varepsilon}}x+D\sin\sqrt{\frac{\lambda}{1+\varepsilon}}x$$

ko'rinishga ega, bu yerda A,B,C va D – ixtiyoriy o'zgarmas sonlar. U holda

$$F'(x)=\sqrt{\frac{\lambda}{1-\varepsilon}}\left(-A\sin\sqrt{\frac{\lambda}{1-\varepsilon}}x+B\cos\sqrt{\frac{\lambda}{1-\varepsilon}}x\right)$$

va

$$G'(x)=\sqrt{\frac{\lambda}{1+\varepsilon}}\left(-A\sin\sqrt{\frac{\lambda}{1+\varepsilon}}x+B\cos\sqrt{\frac{\lambda}{1+\varepsilon}}x\right)$$

bo'lgani uchun, $F'(-1)=F'(1)=0$ va $G'(-1)=G'(1)=0$ chegaraviy shartlarga ko'ra, $B=0,C=0$ bo'lgani uchun, (7)-(8) spektral masalaning xos qiymatlari

$$\lambda_{k,1}=(1+\varepsilon)\left(k+\frac{1}{2}\right)^2\pi^2\ ,\ k\in N,\quad \lambda_{k,2}=(1-\varepsilon)k^2\pi^2\ ,\ k\in N\bigcup\{0\} \quad (13)$$

ko'rinishga ega bo'ladi va bu xos qiymatlarning mos normalangan xos funksiyalari mos ravishda

$$X_{k,1}=\frac{1}{\sqrt{\pi}}\sin\left(k+\frac{1}{2}\right)x,\ k\in N\ ,\quad X_{k,2}=\frac{1}{\sqrt{\pi}}\cos kx,\ k\in N\bigcup\{0\} \quad (14)$$

ko'rinishga ega ekanligini ko'rishimiz mumkin.

(12) va (14) funksiyalar sistemasi $L_2(-1,1)$ fazoda to'la ortonormal sistema tashkil qilishini ko'rsatishimiz mumkin.

Natijada (1),(2) masalaning umumiy yechimi:

$$u(x,t)=\sum_{k=0}^{\infty}C_k e^{\left(k+\frac{1}{2}\right)^2\pi^2(1-\varepsilon)t}\cos\left(k+\frac{1}{2}\right)x+\sum_{k=1}^{\infty}D_k e^{-k^2\pi^2(1+\varepsilon)t}\sin kx$$

(1),(3) masalaning umumiy yechimi:

$$u(x,t)=\sum_{k=1}^{\infty}C_k e^{-\left(k+\frac{1}{2}\right)^2\pi^2(1+\varepsilon)t}\sin\left(k+\frac{1}{2}\right)x+\sum_{k=0}^{\infty}D_k e^{k^2(1-\varepsilon)t}\cos kx$$

ko'rinishda ifodalanishi mumkin.

Shu bilan birga spektral masalalarda (5) tenglamani davriy va antidavriy shartlar bilan qarash mumkin. Masalan, davriy shartlar bo'yicha

$$X''(x)-\varepsilon X''(-x)+\lambda X(x)=0, \tag{15}$$

$$X(-1)=X(1),\ X'(-1)=X'(1) \tag{16}$$

va

$$X''(x)-\varepsilon X''(-x)+\lambda X(x)=0, \tag{17}$$

$$X(-1)+X(1)=0,\ X'(-1)+X'(1)=0 \tag{18}$$

spektral masalalarga ega bo'lamiz.

(15)- (16) spektral masalaning xos qiymatlari

$$(\lambda_{k1})^2=-(1+\alpha)k^2\pi^2\ ,\ (\lambda_{k2})^2=(1-\alpha)k^2\pi^2 \tag{19}$$

tengliklar bilan beriladi. Bu xos qiymatlar oddiy bo'lib, ularga

$$u_{k1}(x)=\sin k\pi x,\ k=0,1,2,....\ ,\ u_{k2}(x)=\cos k\pi x,\ k=0,1,2,.... \tag{20}$$

xos funksiyalar mos keladi.

Xuddi shu kabi (17)-(18) spektral masala xos qiymatlarining ikkita

$$\begin{aligned}(\lambda_{k1})^2&=(1-\alpha)\left(\frac{\pi}{2}+k\pi\right),\ k=0,1,2,...\\ (\lambda_{k2})^2&=(-1-\alpha)\left(\frac{\pi}{2}+k\pi\right),\ k=0,1,2,...\end{aligned} \tag{21}$$

seriyasini topamiz.

Ularga mos xos normalangan funksiyalar

$$\begin{aligned}u_{k1}(x)&=\frac{1}{\sqrt{\pi}}\cos\left(\frac{\pi}{2}+k\pi\right)x,\ k=1,2,....\ ,\\ u_{k2}(x)&=\frac{1}{\sqrt{\pi}}\sin\left(\frac{\pi}{2}+k\pi\right)x,\ k=0,1,2,....\end{aligned} \tag{22}$$

tengliklar bilan aniqlanadi.

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